

Suppose we have a minimization problem

$$\min Z = C_1 x_1 + C_2 x_2 + C_3 x_3 + \dots + C_n x_n$$

subject to some conditions.

Then we can recast it as maximization problem as.

$$\text{maximize } Z' = -C_1 x_1 - C_2 x_2 - \dots - C_n x_n$$

subject to same conditions.

A NEW PROBLEM.

Suppose we have a problem:

$$\text{maximize } Z = 3x_1 + 2x_2 + 5x_3$$

$$\text{subject to } 2x_1 - 3x_2 \leq 3$$

$$4x_1 + 2x_2 - 4x_3 \geq 5$$

$$2x_1 + 3x_3 \leq 2$$

$x_1, x_2 \geq 0$ and x_3 's unrestricted,

Reduce the problem to standard form

Before proceeding to solution we note that x_3 is unrestricted in sign. But all the problems we have done before all the variables were greater than 0 i.e. all the variables satisfy non-negativity condition. In this case we can reformulate the problem by to have all variables satisfying non-negativity condition.

Solution: ~~First we~~ Since x_3 is unrestricted in sign so we replace x_3 by $x_3 = x_3' - x_3''$ where $x_3', x_3'' \geq 0$ so x_3', x_3'' satisfy non-negativity condition. [When $x_3' \geq x_3''$ we have $-x_3 \geq 0$ when $x_3'' \geq x_3'$ we have $x_3 \leq 0$ so we can have unrestricted x_3]

We reformulate the problem as.

$$\begin{aligned} \text{maximize } Z &= 3x_1 + 2x_2 + 5(x_3' - x_3'') \\ &= 3x_1 + 2x_2 + 5x_3' - 5x_3'' \end{aligned}$$

$$\begin{aligned} \text{subject to } 2x_1 - 3x_2 &\leq 3 \\ 4x_1 + 2x_2 - 4x_3' + 4x_3'' &\geq 5 \\ 2x_1 + 3x_3' - 3x_3'' &\leq 2 \\ x_1, x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

Now introducing slack and surplus variable. we can write in standard form as--

$$\text{maximize } Z = 3x_1 + 2x_2 + 5x_3' - 5x_3'' + 0x_4 + 0x_5 + 0x_6$$

$$\begin{aligned} \text{subject to } 2x_1 - 3x_2 + x_4 &= 3 \\ 4x_1 + 2x_2 - 4x_3' + 4x_3'' - x_5 &= 5 \\ 2x_1 + 3x_3' - 3x_3'' + x_6 &= 2 \\ x_1, x_2, x_3', x_3'', x_4, x_5, x_6 &\geq 0 \end{aligned}$$

x_4, x_6 slack variable, x_5 surplus variable.